The revival of target zone modeling

Pompeo Della Posta

Dipartimento di Economia e Management

Via Ridolfi 10

56124 Pisa, Italy

E- Mail: pompeo.della.posta@unipi.it

Abstract

Krugman's 'honeymoon' seminal paper showed in 1991 the stabilizing properties of the adoption of floating bands and gave rise to the literature on target zones. Subsequent contributions, however, showed that, in the absence of adequate reserve, a destabilizing 'divorce' would lead to speculative attacks.

The euro area public debt crisis revived that literature, producing a second generation of target zone modeling, based on the idea that the sustainability of public debt implies the adoption of an interest rate target. Its lack of credibility (if the central bank or the government does not guarantee repayment of the debt) explains the non-linearity of interest rates that was observed during the euro area crisis. The model also allows to understand why some non-euro area countries were not subject to speculative attacks despite the fact that their public debt-to-GDP ratios were as high as those of countries in crisis, but guaranteed by the "virtual" reserves of their national central banks. Finally, a third generation of target zone modeling could also be identified with its application to the current crisis of economic globalization and has been extended to merge it with heterogeneous agents modeling.

Keywords: Target zones, exchange rates, economic fundamentals, speculative attacks, interest rates, public debt, euro area crisis, economic globalization, self-fulfilling expectations

JEL Classification: E65, F34, F36

1. Introduction

The celebrated 'honeymoon' model of exchange rate target zones, presented by Paul Krugman at the end of the 1980s and published eventually in 1991 (Krugman, 1991), generated a vast amount of literature on refinements of the same basic model, although relative to the same case of exchange rates.[1] It proved that the imposition of credible floating bands could stabilize the exchange rate within the target zone, thanks to the beneficial effect of the expected marginal intervention by the central bank: the more the exchange rate moves towards the upper edge of the band, then, the more it would be expected to be pushed back by the reflecting barrier within which it is allowed to fluctuate. As a result, economic fundamentals would enjoy what Krugman (1991) dubbed as a 'honeymoon'.[2] The main thrust of the paper, namely the stabilizing property of a target zone, however, was soon disproved by Bertola and Caballero (1992). They showed that the adoption of a target zone for the exchange rate might generate a 'divorce' rather than a 'honeymoon' if the defense of the margins of the band is not perfectly credible.

What was still unclear, however, was the underlying reason for which a band could be credible or not. An answer to this question came from Krugman and Rotemberg (1992), who made explicit the role played by foreign reserves in guaranteeing the stability of the exchange rate, thereby providing a bridge between the target zone literature and the one on speculative attacks on fixed exchange rates.

The approach of target zone modeling, however, can be extended fruitfully to other domains.

Della Posta (2018 and 2019), for example, shows that the exchange rates target zone model has a straightforward application to the different case of an interest rate target aimed at preventing speculative attacks against public debt, and uses this approach to interpret the recent 2010-2012 euro area crisis. More precisely, such a second generation of target zone models, is applied this time to interest rates (or primary surplus) rather than exchange rates, and considers public debt as the underlying state of economic fundamentals, rather than money supply, while maintaining the same structure and the same economic intuition as in the first generation.[3]

---

[1] Krugman and Miller (1992) contains a first set of important contributions. Kempa and Nelles (1999) provide an accurate theoretical and empirical overview of the large body of literature that developed approximately during the first decade of life of the exchange rate target zone literature. Duarte et al. (2013), instead, reviewed, together with the basic setup of the model, also the subsequent theoretical developments, until about 10 years ago. No significant contributions or applications have appeared after then. This is why I have used the term 'revival' in my paper.

[2] One can think of the state of economic fundamentals as represented, for example, by money supply in a standard monetary model of exchange rate determination.

[3] Duarte et al. (2004) already introduce the distinction between a 'first' and a 'second generation' of target zone modelling. In their terminology, the former is the one relative to the full-credibility 'honeymoon' case, while with the latter they refer to the imperfect-credibility case of 'divorce'.

Finally, a third generation can be identified in the application of the target zone modeling strategy to the current crisis of economic globalization (Della Posta, 2020a, 2020b). In such a case the targeted variable becomes the net cost resulting from economic globalization and the latter is the underlying state of economic fundamentals affecting the former. Such an approach allows to interpret both the pro-globalization climate that has been characterizing the initial years of the latest phase of globalization, and the current phase of retreat of the latter.

This paper surveys the different contributions across the three generations of target zone modeling summarized above in order to outline the theoretical developments that have accompanied its evolution and the flexibility of such a modeling strategy in addressing different issues and in being applied to different environments and it is organized as follows. Section 2 presents the seminal exchange rate target zone literature (the first generation of target zone modeling). Krugman's exchange rate target zone model is presented in Section 2.1 and its 'honeymoon' solution is presented in Section 2.2. Bertola and Caballero's exogenous 'divorce' case is discussed in Section 2.3, and Krugman and Rotemberg's endogenization of the credibility of the target zones, based on the availability of foreign reserves is reviewed in Section 2.4.

Section 3 deals with what can be defined as a second generation of target zone modeling and shows how the basic structure of Krugman's model can be applied to interest rates rather than exchange rates (Section 3.1), so as to obtain a similar 'honeymoon' result (Section 3.2). Section 3.3 considers the possibility of a 'divorce' effect, namely a speculative attack on public debt, rather than on exchange rates and Section 3.4 endogenizes it by highlighting the role played by the availability of 'virtual' stabilizing reserves. Section 4 refers to a third generation of target zone models, applied to the case of the current retreat of globalization. Section 5 contains some concluding remarks.

2. The first generation of target zone modeling

2.1 Krugman's exchange rate target zone model

Krugman (1991) shows the stabilizing effect of the imposition of an exchange rates target zone. In order to generalize his contribution, let us consider a standard uncovered interest rate arbitrage equation:

(1) $$i_t = i^* + \frac{E(dS_t)/dt}{S_t}$$

Where $E(dS_t)/dt$ is the instantaneous expected variation of the exchange rate, S_t. Equilibrium on the monetary market is obtained by imposing the equality between the real money supply $(\frac{M_t}{P_t})$ and the money demand $(\frac{V_t^{\alpha_0}}{i_t^{\alpha_1}})$. The latter is assumed to depend directly on a transactional component including real income, velocity etc., as represented by the term V_t, which is weighted by its elasticity, α_0, and inversely by a speculative component, represented by the interest rate i_t, which is also weighted by its elasticity, α_1:

(2) $$\frac{M_t}{P_t} = \frac{V_t^{\alpha_0}}{i_t^{\alpha_1}}.$$

Purchasing power parity is also assumed to hold:

(3) $$P_t = S_t P_t^*$$

In order to simplify the model, we assume $V_t = 1$ (or, equivalently, $\alpha_0 = 0$), $P_t^* = 1$ and $i_t^* = 0$. By taking logs, it follows $m_t = s_t - \alpha_1[\frac{E(ds_t)}{dt} - s_t]$, that is:

(4) $$s_t = \alpha m_t + \beta \frac{E(ds_t)}{dt},$$

Where $\alpha = \frac{1}{1+\alpha_1}$, $\beta = \frac{\alpha_1}{1+\alpha_1}$, and small letter variables are the logs of capital letter variables.[4]

Eq. (4) says that the exchange rate moves linearly with the money supply and it is also affected by its future expected instantaneous variation, with a weight given by β and ranging between 0 and 1.

The exchange rate is assumed to evolve following a Brownian (or Wiener) motion:

(5) $$dm_t = \sigma dz$$

Where dz is the variation of a Wiener process which si characterized as follows:

(6) $$dz = \chi\sqrt{dt},$$

with χ being an identical, independent and normally distributed random variable with $\mathrm{E}(\chi) = 0$ and $E[\chi - \mathrm{E}(\chi)]^2 = 1$, and dt being an infinitesimal instantaneous time variation. σ is the instantaneous standard deviation of the Brownian motion.

Given the imposition of an exchange rates target zone, it is assumed that:

(7) $$s_t = \bar{s} \text{ if } s_t \geq \bar{s}$$
$$s_t = \tilde{s}_t \text{ if } \underline{s} < s_t < \bar{s}$$
$$s_t = \underline{s} \text{ if } s_t < \underline{s},$$

[4] Krugman's model assumes instead the exchange rate equation: $s_t = m_t + \beta \frac{E(ds_t)}{dt}$.

where $\bar{s}$, $\underline{s}$, $\tilde{s}_t$ and s_t represent respectively the upper and the lower thresholds for the exchange rate, the exchange rate that would obtain when it fluctuates within the announced bands, and the exchange rate prevailing in case no commitment is taken by the central banks.

To summarize, then, Krugman's exchange rate target zone model can be summarized by Eqs. (4), (5), (6) and (7).

2.2 Solving for Krugman's 'honeymoon' effect

In order to calculate the value of the exchange rate within the floating band, let us consider a generic functional form for the exchange rate, that will be used to obtain a closed form solution, given the assumptions for the stochastic process that regulates domestic credit creation, that is Eqs. (5) and (6). In particular, let us assume that the exchange rate is a function of the money supply:

(8) $$s_t = g(m_t)$$

To simplify the notation and to make it uniform to what will follow in the next sections, let us omit time subscripts, as it is usually done in this literature. In order to determine the exchange rate variation, let's calculate Ito's differential:

(9) $$ds = g'(m)(dm) + \frac{1}{2}g''(m)(dm)^2$$

From the definition of dm as given in (5), we have that $(dm)^2 = \sigma^2\chi^2 dt$. Considering in (9) expected values and dividing by dt (and knowing that $E(dm)/dt = 0$, and $\frac{E(dm)^2}{dt} = \sigma^2$), we obtain Ito's Lemma:

(10) $$\frac{E(ds)}{dt} = \frac{1}{2}g''(m)\sigma^2$$

By replacing (10) into (4) we have, then:

(11) $$s_t = g(m) = \alpha m + \beta\frac{\sigma^2}{2}g''(m).$$

This is a second order differential equation whose generic solution is:

(12) $$s = g(m) = \alpha m + Ae^{\lambda m}$$

By calculating the second order derivative it follows that:

(13) $$g''(m) = \lambda^2 Ae^{\lambda m},$$

that, replaced in (11) will give:

(14) $$s = \alpha m + \beta \frac{\sigma^2}{2}(\lambda^2 A e^{\lambda m})$$

By comparing (14) with (12), we have:

(15) $$A e^{\lambda m}\left(\beta \frac{\sigma^2 \lambda^2}{2} - 1\right) = 0$$

Constant *A* will be determined thanks to the introduction of an initial condition or, as we will see, a final condition. The solution will be given, then, by the values of λ satisfying the characteristic equation $\left(\beta \frac{\sigma^2 \lambda^2}{2} - 1 = 0\right)$, from which it follows that:

(16) $$\lambda_{1,2} = \pm\sqrt{\frac{2}{\beta\sigma^2}}.$$

This means that there exist two complementary solutions satisfying our second order differential equation, namely $s_t^{c1} = A_1 e^{\lambda_1 m}$ and $s_t^{c2} = A_2 e^{\lambda_2 m}$, that we are going to add to each other so as to obtain the general solution:

(17) $$\tilde{s} = g(m) = \alpha m + A_1 e^{\lambda_1 m} + A_2 e^{\lambda_2 m},$$

with $\lambda_1$and λ_2 defined as in (16).

Assuming the presence of the upper band only, we can ignore A_2. In the lower band the exchange rate behaves like in a perfect free float.

The solution of the model is obtained by the so called 'smooth pasting' closing condition, according to which the path of the exchange rate within the band will be tangent to the level of the fixed exchange rate. Intuitively, one can think that the more the exchange rate approaches its upper target, the higher the probability that it will be pushed back within the band. This obtains thanks to a tangency condition on the horizontal axis.

When $\tilde{s}$ reaches $\bar{s}$ – in correspondence with the state of fundamentals m' - it will be, then (still assuming $A_2 = 0$), that:

$$\frac{d\tilde{s}}{dm'} = \alpha + \lambda_1 A e^{\lambda_1 m'} = 0,$$

from which it follows that:

(18) $$A = -\frac{\alpha}{\lambda_1} e^{-\lambda_1 m'} < 0.$$

By replacing (18) into (17) (still with $A_2 = 0$ and calculated when $m = m'$), we have that: $\tilde{s}(m') = \bar{s} = \alpha m' - \frac{\alpha}{\lambda_1}$, that is:

(19) $$\tilde{s}(m') = \bar{s} = \alpha m' - \alpha\sqrt{\frac{\beta\sigma^2}{2}}$$

In the case of a free float (*FF*), in which no exchange rate change is expected because no central bank intervention is anticipated, the non-linear part of the above equation drops and we have that $\bar{s} = \alpha\bar{m}$, implying that the value of the exchange rate $\bar{s}$ will be reached when the fundamental variable reaches the value $\bar{m}$.

The difference between m' and $\bar{m}$, respectively the level of money supply reached in the case of a target zone and in the case of a free float when the upper exchange rate target level $\bar{s}$ is reached, turns out to be (also recalling Eq. 16):

(20) $$m' - \bar{m} = \sqrt{\frac{\beta\sigma^2}{2}} = \frac{1}{\lambda}.$$

This is what Krugman dubbed as the 'honeymoon' effect and indicates the measure by which it is possible to expand money supply while remaining within the exchange rate floating band (see Figure 1, where superscript *H* refers to the 'honeymoon' effect).

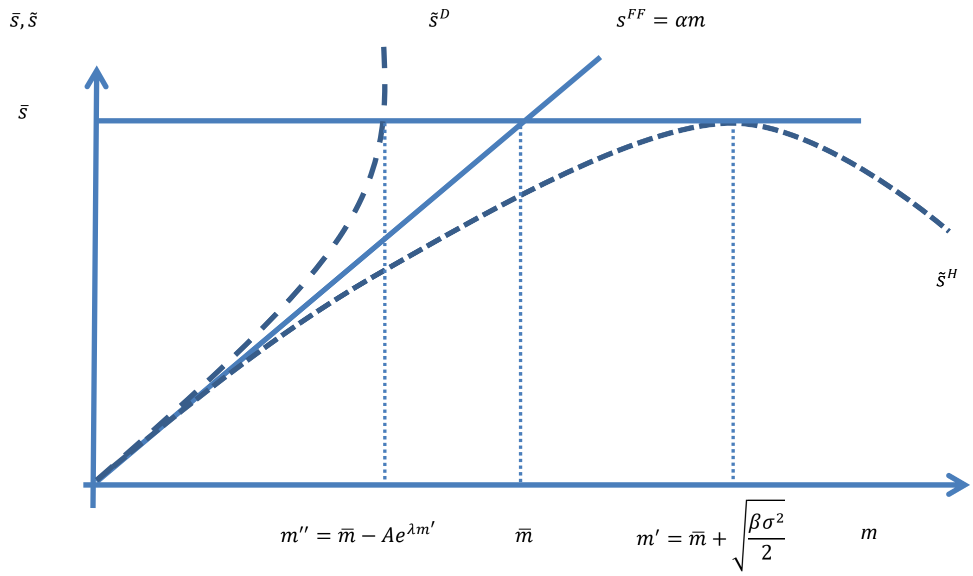

Figure 1: The 'honeymoon' and 'divorce' effects in the exchange rate target zone literature.

It should be noticed that parameter β stabilizes the exchange rate by enlarging the size of the 'honeymoon'. As a matter of fact, speculation will anticipate the marginal intervention of the central bank preventing the exchange rate to move beyond it. Moreover, the larger the volatility of the economic fundamentals, the larger the probability that the exchange rate, moving towards the top of the band, will be pushed back by the marginal intervention of the monetary authorities.

2.3 Bertola and Caballero's exogenous 'divorce' effect

In spite of the elegance of the result obtained by Krugman (1991), it was argued soon that the indication of a reference target for the exchange rate could produce a destabilizing, rather than stabilizing effect. Bertola and Caballero (1992) showed that the adoption of a target zone for the exchange rate might generate a 'divorce' rather than a 'honeymoon' if the defense of the margins of the band was not credible. The 'smooth pasting' solution would only apply in the case in which there is full confidence that the exchange rate will not be allowed to increase above its upper target. When that is not the case, instead, the more the exchange rate moves towards its upper margin, the higher the expectation of a devaluation. This is the case in which the 'divorce' emerges.

Money supply can be assumed as fluctuating between 0 and the maximum level ($\bar{m}$) which is obtained in correspondence with the upper target for the exchange rate, while the center of the band of the economic fundamental can be taken as equal to $\bar{m}/2$.

An arbitrage argument provides the closing equation. When the exchange rate reaches its upper threshold, $\bar{s}$, its value has to be equal to the expected one resulting from the weighted probabilities of the two different events that may take place. There is a probability *p* that it will not be possible to prevent the money supply to exceed the maximum level compatible with a fixed exchange rate, so that the latter will be expected to be devalued. So, one possibility is that when *m* reaches $\bar{m}$, it will be allowed to move up by a band of the same size $\bar{m} > 0$ (an assumption that has been removed and generalized by Della Posta, 2019) and it will jump to the center of the new fluctuation band, namely to $\bar{m} + \bar{m}/2$, which is included between $\bar{m}$ and $2\bar{m}$.

Of course, there is also the complementary probability (1-*p*) that money supply will not be allowed to increase, by keeping it at or below $\bar{m}$. The adjustment at the margin might be such as to move its floating band down by $\bar{m}$ (again, an assumption that can be generalized) and the money supply will go back to the center of a the old floating band, $\bar{m} - \frac{\bar{m}}{2}$, which is included between 0 and $\bar{m}$.

It turns out, then, that the arbitrage equation is as follows:

$$p\,\tilde{s}\left(\bar{m}+\frac{\bar{m}}{2},\bar{m}+\frac{\bar{m}}{2}\right)+(1-p)\tilde{s}\left(\bar{m}-\frac{\bar{m}}{2},\bar{m}-\frac{\bar{m}}{2}\right)=\tilde{s}\left(\bar{m},\frac{\bar{m}}{2}\right),$$

Namely:

(21) $$p\ \tilde{s}\left(\frac{3}{2}\bar{m},\frac{3}{2}\ \bar{m}\right)+(1-p)\tilde{s}\left(\frac{1}{2}\bar{m},\frac{1}{2}\bar{m}\right)=\tilde{s}\left(\bar{m},\frac{\bar{m}}{2}\right),$$

Where in $\tilde{s}(m,c), m$ refers to the current value taken by the fundamental, and *c* refers to the value taken by the fundamental at the center of the band. By considering only the upper band, as it was done above in order to solve for the 'smooth pasting' case, we have that:

(22) $$\tilde{s}(m,c)=\alpha m+Ae^{\lambda(m-c)}.$$

Replacing (22) into (21), we have that:

(23) $$p[\alpha\left(\frac{3}{2}\bar{m}\right)+A]+(1-p)\ [\ \alpha\left(\frac{1}{2}\bar{m}\right)+A]=\alpha\bar{m}+Ae^{\lambda\frac{\bar{m}}{2}},$$

from which it follows that:

(24) $$A=\frac{[p(\alpha\bar{m})-\frac{\alpha\bar{m}}{2}]}{e^{\lambda\frac{\bar{m}}{2}}-1}.$$

This also means that $A\geq 0$ iff $[p(\bar{m})-\frac{\bar{m}}{2}]\ \geq 0$, that is iff:

(25) $$p\ \geq\frac{1}{2}.$$

In that case, then, we will have that $\tilde{s}^{D}=\alpha m+Ae^{\lambda m}>s^{FF}=\ \alpha m$, where $\tilde{s}^{D}$ is the value of the exchange rate when a non-credible 'divorce'-type target zone is applied and where s^{FF} is the value of the exchange rate in the case of free float.

In that case, then, considering again that in the *FF* case $\bar{s}=\alpha\bar{m}$ and that in the case of 'divorce' the upper band will be reached when m reaches the value m'', we will have that in the *TZ* case: $\tilde{s}^{D}(m'')=\bar{s}=\alpha m''+Ae^{\lambda m''}$. The intuition for this 'divorce' result is quite straightforward: it is sufficient that the probability of an exchange devaluation be larger than the probability of a defense (assuming the new band to be as large as the initial one), to produce a destabilizing convex trajectory for the exchange rate when approaching the upper threshold. Of course, the opposite would hold for the lower band.

The result obtained above confirms the convex non-linearity of the exchange rate behavior, as driven by the expectation of a devaluation, rather than by the defense of the upper target.

Figure 1 above represents in a simplified way what has just been described analytically (with superscript *D* referring to the 'divorce' effect), so that $m''<\bar{m}$.

2.4 Krugman and Rotemberg's endogenization of the 'divorce' effect

2.4.1. A different modeling

While the contribution of Bertola and Caballero (1992) disavowed Krugman's 'honeymoon' result, it still treated exogenously the expectation of a devaluation or defense of the exchange rate, leaving unexplained what determined one state of expectations or the other.

This is a weakness that Krugman e Rotemberg (1992) fixed, by identifying the linkage between target zone and speculative attacks literature, and therefore by endogenizing the choice of devaluing or defending the exchange rate target.

Let us consider a model which is the same as the one that was presented at the beginning, but in which the stochastic money demand shock, V_t, is not taken as constant and equal to 1 anymore (to simplify the notation, I omit again time indexes, as Krugman and Rotemberg, 1992, also do):

(26) $$\frac{M}{P} = \frac{V^{\alpha_0}}{I^{\alpha_1}}$$

This allows us (by assuming $\alpha_0 = 1$ and by considering again also Eqs. (1) and (3) above), to derive the structure of the log-linear equation which is used by Krugman and Rotemberg (1992), and whose only difference with respect to the one from which we started our analysis in Section 1 is represented by the fact that the money demand shock V is now assumed to follow a Brownian motion, while the nominal money supply M is assumed to be constant. In other words, the economic fundamental variable that is assumed to affect the exchange rate, rather than being the money supply, becomes its excess over money demand. By taking logs we have the following log-linear equation, where it is still the case that $\alpha = \frac{1}{1+\alpha_1}$, $\beta = \frac{\alpha_1}{1+\alpha_1}$, and small letter variables are the logs of capital letter variables, as in Section 1:

(27) $$s = \alpha(m + v) + \beta \frac{E(ds)}{dt}$$

The money demand shock increasing the excess of money supply is assumed to follow the Brownian motion already considered in Eqs. (5) and (6) of the initial model. I assume, however, a driftless motion, contrary to what Krugman and Rotenberg (1992) do, because including an exogenous drift would not add any significant element to our discussion:

(28) $$dv = \sigma dz$$

As for dz, the same Brownian motion process assumed in Eq. (6) applies.

To summarize, then, the model to solve is characterized by Eqs. (27), (28), (6) and (7).

2.4.2 The solution procedure

The procedure to follow now in order to solve the model in the case of a credible target zone is absolutely the same as the one used above in Eqs. (8) and following, and leads us to the same results, the only difference being that now we have to start by assuming the exchange rate *s* as determined by a generic function *g* which depends on both *m* and *v*: $s = g(m + v)$. The solution of the resulting equation is:

(29) $$\tilde{s} = \alpha(m + v) + Ae^{\lambda v},$$

from which, following the same steps taken in Eq. (8) and following, for the case of a credible target zone it still turns out that:

(30) $$\lambda_{1,2} = \pm\sqrt{\frac{2}{\beta\sigma^2}} > 0.$$

Target zones, however, may have different degrees of credibility, as I am going to show below.

Closing the model in the case of a target zone with no reserves.

If, the central bank does not intervene in the foreign exchange market, nor she is expected to do so, the exchange rate floats freely. As a result, the level of foreign reserves and of the money supply remains unchanged, the target zone plays no role and the freely floating exchange rate, then, follows the initial *FF* line in Figure 2, characterized by $A = 0$:

(31) $$s^{FF} = \alpha(m + v)$$

When an upper target is adopted, instead, what matters is the level of foreign reserves that are available and that allow to intervene in defense of the exchange rate parity, as it will be discussed in the next two paragraphs.

Closing the model in the case of target zones with limited reserves.

Money supply is composed by the stock of domestic credit and foreign reserves. It follows that:

(32) $$m = \ln(D + R)$$

A successful speculative attack wipes out the stock of available foreign reserves (let us assume that the minimum level of reserves, at which the exchange rate is left free to float, is zero). It follows, then, that:

(33) $$m' = \ln(D)$$

As a result, after the attack, the floating exchange rate will reach the $F'F'$ line in Figure 2 whose generic equation is given by $s^{F'F'} = \alpha(m' + v)$. In *C*, when the shock takes the value v^{PC}, then, the exchange rate will take the value:

(34) $$\bar{s} = \alpha(m' + v^{PC}),$$

where superscript *PC* refers to the case of partial credibility.

What needs to be done is to calculate the trajectory within the band, so that $\tilde{s}$ will reach the $F'F'$ line (the one characterized by the remaining lower level of money supply, m'), exactly when the money demand shock takes the value v^{PC}. This would guarantee the absence of discrete exchange rate jumps that would violate otherwise the no-arbitrage condition. In order to find *A* in Eq. (26), then, it must be the case that when the money demand shock takes value v^{PC}, the exchange rate moving within the band, $\tilde{s}$, reaches its upper margin, $\bar{s}$.

The exchange rate prevailing within the band when the shock takes value v^{PC}, therefore, will be:

(35) $$\tilde{s}^{PC} = \alpha(m + v^{PC}) + Ae^{\lambda v^{PC}} = \bar{s},$$

(where $\tilde{s}^{PC}$ is the exchange rate moving within a partially credible *TZ*), while the one prevailing after the attack and the central bank's response is described by (Eq. 34).

By equating (35) with (34) we have that:

$$m' = m + Ae^{\lambda v^{PC}},$$

namely:

(36) $$\frac{m' - m}{e^{\lambda v^{PC}}} = A < 0$$

It turns out that the constant A is negative, given that $m > m'$. Markets' expectation of central bank intervention strengthens the exchange rate and avoids the depreciation that would occur with flexible exchange rates. Money supply remains constant at level m within the band, but at point *D* in Figure 2 the speculative attack occurs, m is reduced to m', foreign reserves are exhausted, and the exchange rate starts floating. In this case, as Krugman and Rotemberg (1992) observed, the 'smooth pasting' condition plays no role: the exchange rate will touch the upper band without any tangency condition but with what could be defined as a 'hard hitting' instead (see the curve $\tilde{s}^{PC}$ in Figure 2).

Closing the model in the case of target zones with large reserves

Eq. (36) above suggests that the larger the amount of reserves (as measured by the difference between m and m'), the larger the absolute value of *A*, and the larger the flattening of the exchange rate curve within the band. 'Smooth pasting' emerges, then, as a solution of the model when foreign reserves are large enough to guarantee the defense of the target zone. The condition that allows to close the model and identify *A* in the case in which the target will be defended, then, will be precisely the 'smooth pasting':

$$\frac{d\tilde{s}^C}{dv^C} = 1 + \lambda A e^{\lambda v^C} = 0$$

i.e.:

(37) $$\frac{1}{\lambda} = -Ae^{\lambda v^C},$$

Where $\tilde{s}^C$ is what I had called $\tilde{s}^H$ when discussing the seminal Krugman's model, namely the exchange rate moving within a credible target zone and v^C is the money demand shock at which $\tilde{s}^C = \bar{s}$.

At the upper limit of the band, then, by following the $\tilde{s}^C$ curve in Figure 2, we will have that:

(38) $$\tilde{s}^C = \bar{s} = \alpha(m + v^C) - \frac{\alpha}{\lambda}.$$

We also know that after the attack the exchange rate will have to be on the $F''F''$ line:

(39) $$\bar{s} = \alpha(m' + v^C)$$

So that, by equating the two previous equations we have:

(40) $$m' - m = -\frac{1}{\lambda}$$

i.e., considering the definitions of m' and m given above – notice that the novelty of the approach of Krugman and Rotemberg (1992) consists of just this enlightening insight – it turns out that the amount of reserves that are available covers exactly the size of the 'honeymoon' ($\frac{1}{\lambda}$) emerged by imposing the 'smooth pasting':

$$\ln(D + R) - \ln(D) = \frac{1}{\lambda}$$

By calculating the anti-logarithm, we have that:

$$1 + \frac{R}{D} = e^{1/\lambda}$$

Which means that the 'smooth pasting' will obtain if:

(41) $$\frac{R}{D} \geq e^{1/\lambda} - 1.$$

As long as the foreign reserves are large (in particular, as long as $R \geq (e^{1/\lambda} - 1)D$), then, $\bar{s}$ will work as a reflecting, rather than absorbing barrier: when the exchange rate will reach the upper margin it will bounce back within the band.

Of course, this is something that cannot last forever. Central bank intervention will shift gradually the exchange rate schedule to the right. The larger the realizations of the shocks, however, the larger the loss of foreign reserves and the situation will move from one with lots of reserves and 'smooth pasting' to one with limited reserves and 'hard hitting', implying inevitably that the defense of the exchange rate will have to be abandoned and it will start floating (Flood and Garber, 1991).

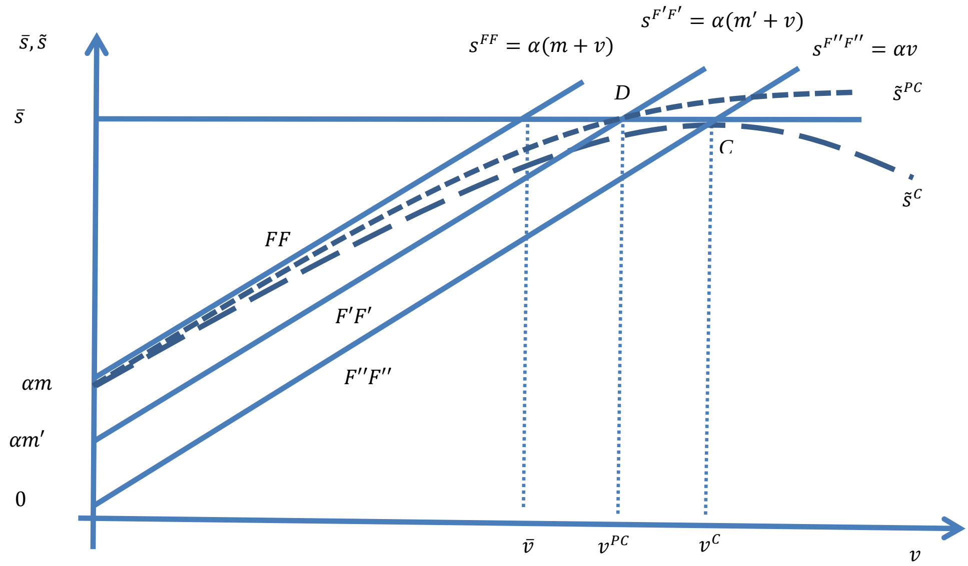

Figure 2: A partially credible (PC) and a fully credible (C) exchange rate target zone.

2.4.3 An extension of Krugman and Rotemberg's model in the case of a negative self-fulfilling prophecy: a convex non-linearity for the dynamics of the exchange rate

Krugman and Rotemberg (1992) stressed the role played by the full availability of foreign reserves in explaining smooth pasting and showed also how some limited reserves would determine a correspondingly limited credibility and, as a result, a lower size of the 'honeymoon'.

In their model they did not obtain, however, the case of a convex non-linearity of the dynamics of the exchange rate, the so-called 'divorce'- which is the opposite of the 'honeymoon' - as Bertola and Caballero (1992) dubbed it. Such a conclusion obtains in the case of negative self-fulfilling prophecies, in which the pegging of a fixed exchange rate is expected not to hold, a devaluation takes place and, following it, monetary policy increases, thereby validating it ex-post (Obstfeld, 1986).

Considering, as done by Della Posta (2018), a one-sided-only band we have that $\tilde{s}^X = \bar{s}$ when v takes the value v^X (with which I indicate the size of the monetary policy shock at which the exchange rate, $\tilde{s}^X$ reaches its upper level, $\bar{e}$, in the case in which money supply is expected to increase):

(42) $$\tilde{s}^X = \bar{s} = \alpha(m + v^X) + Ae^{\lambda v^X}$$

If we know that the central bank does not have enough foreign reserves to avoid an exchange rate devaluation, so that following it a money supply spike will take place, we also know that after the speculative attack resulting from the money demand shock, v^X, it will not be possible to target the exchange rate anymore. This means that the latter will be moving along the line $F'F'$, and when the money demand shock v^X occurs, it will take the value:

(43) $$\bar{s} = \alpha(m^X + v^X),$$

where m^X indicates the expected level of money supply after the abandonment of the exchange rate target zone.

A standard arbitrage argument suggests that there cannot be any discrete jump in the exchange rate when moving from one regime to the other, namely from Eq. (42) to Eq. (43). At point E in Figure 3, then, the two curves need to take the same value, so that it must be that $Ae^{\lambda v^X} = \alpha(m^X - m)$, namely:

(44) $$\frac{\alpha(m^X - m)}{e^{\lambda v^X}} = A > 0.$$

This means that if $m^X > m$, we obtain a novel result, namely that the exchange rate curve within the band will follow a convex non-linearity, given that *A* takes a positive value, exactly as in the case of the 'divorce effect' studied by Bertola and Caballero (1992).

Let us compare now the value of v^X with the value of $\bar{v}$ (see Figure 3).

From the equation of the $F'F'$ line it turns out that:

(45) $$v^X = \frac{\bar{s}}{\alpha} - m^X$$

From the equation of the FF line (which is obtained by considering the exchange rate equation in the absence of any expected intervention to stabilize it and for a given level of money supply (*m*) namely $s^{FF} = \alpha(m + v)$, instead, it turns out that at $\bar{s}$ we have:

(46) $$\bar{v} = \frac{\bar{s}}{\alpha} - m$$

It is easy to see that $\bar{v}$ is the value of the money demand shock at which the target $\bar{s}$ is hit with an initial money supply-to-GDP ratio, m. Using Eqs. (45) and (46), then, we have that:

(47) $$v^X = \bar{v} - (m^X - m) < \bar{v}.$$

This means that the exchange rate reaches its upper target $\overline{m}$ in correspondence of a public debt demand shock which is lower than the one relative to the absence of a target, as with the 'divorce'.

Let us consider now the second generation of target zone modeling, applied to interest rates and allowing to study and interpret the 2010-2012 euro area public debt crisis.

3. The second generation of target zone modeling: interest rate targets and speculative attacks on public debt

Della Posta (2018, 2019) adopts a target zone modeling technique - by considering a target for the interest rate, rather than for the exchange rate - to explain the convex non-linearity of interest rates during the euro area crisis. In doing so he joins together the monetary and the fiscal explanations of the euro area crisis provided respectively by De Grauwe (2012) and De Grauwe and Ji (2013a, 2013b), who focus on the stabilizing role of the presence of a monetary lender of last resort and by Tamborini (2015), who stresses instead the negative effects of unsustainable austerity fiscal policies.

3.1 The setup of the basic interest rate target zone model

The interest rate, i_t on public debt can be thought as determined by an arbitrage equation. We can consider a riskless (foreign) reference interest rate, $\bar{r}$ and a risk premium, ρ_t.

(48) $$i_t = \bar{r} + \rho_t,$$

In turn, the latter depends on two elements. The first one is the absolute size of the public debt-to-GDP ratio (Corsetti *et al.*, 2014), due to the fact that the higher b_t, the lower the potential to respond to negative shocks hitting the economy. The sensitivity of the interest rate with respect to b_t is measured by parameter α. The second part, instead, is characterized by self-fulfilling features. The lower the expected sustainability of public debt, the lower the price that investors would be expected to be willing to pay for it, and the higher the expected future variation of the interest rate. In turn, the latter affects the current interest rate level with a weight given by parameter β. This is represented in Equation (49) below:

(49) $$\rho_t = \alpha b_t + \beta \frac{E[di_t]}{dt},$$

Assuming, in order to simplify the model, that $\bar{r} = 0$, Eq. (38) becomes, then (but it would be indifferent to use as dependent variable the interest rate spread, ρ_t rather than the nominal interest rate, i_t):

(50) $$i_t = \alpha b_t + \beta \frac{E(di_t)}{dt}.$$

The value of the interest rate is targeted by central banks, since a too high level would be detrimental to public debt stability and to the economy. In the case of a credible targeting, the target values for i_t can be identified as follows:

$$i_t = \bar{\iota} \text{ if } i_t \geq \bar{\iota}$$

(51) $$i_t = \tilde{\iota}_t \text{ if } i_t < \bar{\iota}$$

where $\bar{\iota}$, $\tilde{\iota}_t$ and i_t represent respectively the upper central bank's thresholds for the interest rate, the interest rate that would obtain when it fluctuates within the announced band, and the interest rate prevailing with free float when no commitment is taken by the central bank (ignoring the lower threshold, as we have already done when analyzing the case of exchange rate target zones).

The standard public debt dynamics, namely the continuous time variation of the public debt-to-GDP ratio, db_t, is as follows:[5]

(52) $$db_t = -(m_t + s_t)dt + (i_t - g_t)b_t dt + \sigma dz.$$

The term s_t is now the primary public surplus-to-GDP ratio, m_t is now the public debt monetization-to-GDP ratio. The term $(i_t - g_t)b_t$ is the interest rate service (net of GDP growth) on the debt-to-GDP ratio. If the deterministic part of the public debt-to-GDP ratio is *stabilized* it must be that:

$$-(m^* + s^*)dt + (i^* - g^*)b^* dt = 0,$$

where starred variables refer to those variables that guarantee that the deterministic part of $db_t = 0$. In the equation above, s^* is the primary surplus-to-GDP ratio that a government should run to stabilize public debt, $m^*$is the monetary growth-to-GDP ratio that the central bank should run to

[5] See Della Posta (2019) for further details on the government debt stability condition.

stabilize public debt, g^* is a long run GDP growth and i^* is the resulting interest rate granting public debt stability, with b^* being the steady state value of public debt.

Such a *stability* condition is only *sustainable*, however, if the fiscal and/or the monetary authority are actually able to operate to make sure that the deterministic part of public debt is stabilized, namely if:

$$-(\bar{m}+\bar{s})dt + (\bar{\imath} - g^*)b^* dt = 0,$$

that can be rewritten as:

(53) $$\bar{\imath} = g^* + \frac{\bar{m}+\bar{s}}{b^*}$$

In the equation above $\bar{m}$ is the largest possible – therefore credible – monetary growth-to-GDP ratio that the central bank can run to stabilize public debt, $\bar{s}$ is the maximum feasible – therefore credible – primary surplus-to-GDP that a government can run to stabilize public debt and $\bar{\imath}$ is the resulting highest possible interest rate granting public debt stability, which also represents the upper threshold of the interest rate target zone seen in Eq. (51) above.[6]

Eq. (53) tells, then – something that it is worth underlining - what are the policy determinants of the upper sustainability threshold on interest rates, and it will play, then, a quite relevant role in discussing the credibility of the interest rate target zone. [7]

When Eq. (53) is satisfied, Eq. (52) becomes:

(54) $$db = \sigma dz.$$

(dropping again time subscripts to simplify the notation). The stochastic component of public debt-to-GDP growth, then, is supposed to follow a Brownian motion process σdz, where σ still represents the instantaneous standard deviation of the Brownian motion and the term dz is the Brownian motion variation which is characterized as in Eq. (6) above.

As soon as i_t exceeds $\bar{\imath}$, then, public debt is not sustainable anymore and this generates an explosive spiral between interest rates and public debt.

If the fiscal authority or the central bank are expected instead to intervene respectively by increasing (feasibly) the primary surplus or by buying public debt, to avoid a drop of the price of bonds, this means that the

[6] See Tamborini (2015) for further details on the conditions that have to hold for public debt sustainability

[7] Eq. (53) also suggests that it would be possible to use as a dependent variable of the target zone model also either m_t or s_t.

interest rate remains within the band ($i_t \leq \bar{\imath}$) and this would produce a 'honeymoon' which is similar to the one identified by Krugman (1991).

The model in the case of an interest rate target zone is therefore composed by Equations (50), (51), (53), (54) and (6).

3.2 Feasibility of central bank's and/or government's intervention: the 'honeymoon'.

By following the steps taken by Krugman (1991) in his seminal paper, in which however the independent variable is now the public debt-to-GDP ratio, b, and the dependent variable is the feasible interest rate, $\tilde{\imath}$, we obtain the 'smooth pasting'-tangency condition, that in this case is: $\frac{d\tilde{\imath}}{db} = 0$, and that gives:

(55) $$b' = \bar{b} + \frac{1}{\lambda_1}$$

Where $\bar{b} = \frac{\bar{\imath}}{\alpha}$ is the stable and sustainable public debt-to-GDP ratio that obtains when the interest rate reaches $\bar{\imath}$ by following its linear path, without resenting of the imposition of the upper band.

The term b' is instead the largest sustainable level that can be reached when considering also the expectational effects produced by the imposition of a credible interest rate target zone.

The difference between b' and $\bar{b}$, which is given by $\frac{1}{\lambda_1} = \sqrt{\frac{\beta\sigma^2}{2}}$, then, is the size of the public debt 'honeymoon' and tells us by how much the public debt-to-GDP ratio can increase beyond the sustainability level obtained in the absence of any expectational effect while keeping $i_t \leq \bar{\imath}$.

What precedes, however, would only apply if the interest rate can be credibly stabilized. When this is not the case a different closing condition needs to be considered.

3.3 An extended application of Bertola and Caballero's exogenous 'divorce' to an interest rate target zone

If the government cannot commit credibly to guarantee public debt solvency and/or the central bank is unavailable to act as a buyer of last resort, the 'smooth pasting' solution cannot apply.
In that situation, when i_t hits the interest rate threshold, $\bar{\imath}$, the former is expected not to be able to stabilize the public debt-to-GDP ratio. This expectation produces destabilizing rather than stabilizing effects. Following Bertola and Caballero (1992) in the different context of an exchange rate target zone,

Della Posta (2019) assumes that the public debt-to-GDP ratio, b, fluctuates between 0 and the maximum level of public debt ($\bar{b}$) which is obtained in correspondence with the maximum feasible interest rate that assures public debt stability ($\bar{\imath}$), while the center of the band of the economic fundamental can be taken as equal to $\bar{b}/2$. Assuming that when the value of public debt-to-GDP ratio reaches $\bar{b}$, it will be allowed with probability *p* to move up by the size $\delta\bar{b}$ and with the complementary probability (1-*p*) to move down by the size $\varepsilon\bar{b}$ and considering a symmetric fluctuation band centered on point *c* and focusing only on the upper band, so that $\tilde{\imath}(b,c) = \alpha b + Ae^{\lambda(b-c)}$, we can conclude that $A \geq 0$ iff $p \geq \frac{\varepsilon}{\delta+\varepsilon}$.

In the 'one-way bet' case in which public debt is only expected to increase (namely the case in which $\varepsilon = 0$) then, a 'divorce' effect will always appear.

It should be observed that the formulation introduced by Della Posta (2019) generalizes the result obtained by Bertola and Caballero (1992), who considered instead the case of the probability of a new upper or downward floating band of equal size.

3.4 An application of Krugman and Rotemberg's endogenous 'divorce' to an interest rate target zone.

As we have seen in Section 2.3 above, Krugman and Rotemberg (1992) endogenized the lack of (or reduced) credibility of the defense of an exchange rates target zone. Della Posta (2018) shows that their approach is easily applicable to the case of an interest rates target zone too, considering the following slightly modified interest rate equation:

(56) $$i_t = \alpha(b_t + v_t) + \beta\frac{E[di_t]}{dt}.$$

To simplify the notation, let us omit again the time indexes, as done above.

The procedure to solve Eq. (56) follows the standard steps that we have seen above (see Della Posta 2018 b, for further details) and uncovers the role played by the 'virtual' reserves that are available to guarantee public debt both with the central bank – if and as much as she can play the role of buyer of last resort standing ready to cover the public debt - and with the fiscal authority - depending on the availability of some extra fiscal space to run a primary surplus in order to stabilize public debt.

Indicating with VR the total amount of such 'virtual' reserves (namely the additional monetary and fiscal space that would be available in order to stabilize the public debt-to-GDP ratio), it is possible to consider three different cases, ranging between the most unstable one, in which the market expects the central bank and the government not to be able to stabilize public debt, and are in fact expecting the latter to increase to a higher level b^X, and the most stable one, in which the central bank [8] and/or the fiscal authority guarantee its full repayment ($VR = b$). The third, intermediate, situation would be one in which the central bank and the fiscal authority jointly can only partly guarantee the repayment of public debt ($VR^{PC} < b$).

3.4.1 A non-credible interest rate target zone with an increasing public debt-to-GDP ratio

In the case in which a breaching of the interest rate target zone is expected, a 'divorce' - namely an interest rate convex non-linearity, as identified by De Grauwe and Ji (2013a) in the case of the euro area crisis - emerges. It can be concluded, then, following Della Posta (2018), that when the public debt demand shock v^X occurs, after the abandonment of the interest rate target zone, the interest rate will be on the line $F'F'$ in Figure (??) and, given that public debt will increase to a known level b^X, it will take the value:

(57) $$i^{F'F'}(v^X) = \bar{\imath} = \alpha(b^X + v^X).$$

The same arbitrage argument used in the case of exchange rate target zone (see Eq. 35) – according to which the value of the interest rate resulting from the equation above should be the same as the one obtained when considering the expectational effect - leads to conclude that *A* takes a positive value, exactly as in the case of the 'divorce effect' discussed above.

3.4.2 Endogenizing the credibility of the interest rate target zone

The 'smooth pasting' emerges instead in the case of a fully credible public debt stability, implying a credible defense of the interest rate target. As mentioned above, this will only be possible if the central bank and the treasury jointly are expected to be endowed with enough fiscal or monetary space (what can be dubbed as 'virtual' reserves, VR) that is sufficient to absorb the public debt demand shock. The term v^C indicates the largest public debt demand shock that can be resisted in the case of a credible interest rate target zone, thereby reducing the initial supply of public debt to 0. In such a case, if the interest rate was not targeted anymore, it would follow the $F''F''$ path, which is shifted downwards compared to the one with $b > 0$:

[8] Normally, an anti-inflationary and independent central bank will avoid printing money when that may cause inflation, but it is difficult to imagine that a monetary authority which is caught between the need to avoid inflation and the need to avoid the bankruptcy of her own country will prefer the second option.

(58) $$i^{F''F''}(v^C) = \bar{\iota} = \alpha v^C.$$

Still using an arbitrage argument, it turns out that this is only possible if:

(59) $$VR = \frac{1}{\lambda} = v^C - \bar{v} = \sqrt{\frac{\beta\sigma^2}{2}} = b$$

The size of the 'honeymoon', then, coincides with the amount of resources that are available either with the central bank or with the government to cover exactly the existing public debt, b, in the worst-case scenario in which it is fully challenged by the market. In other words, the credibility that materializes in the 'honeymoon effect' is not a gift from heaven, but it is something that is precisely tied down by the availability of 'virtual' reserves, as defined above.

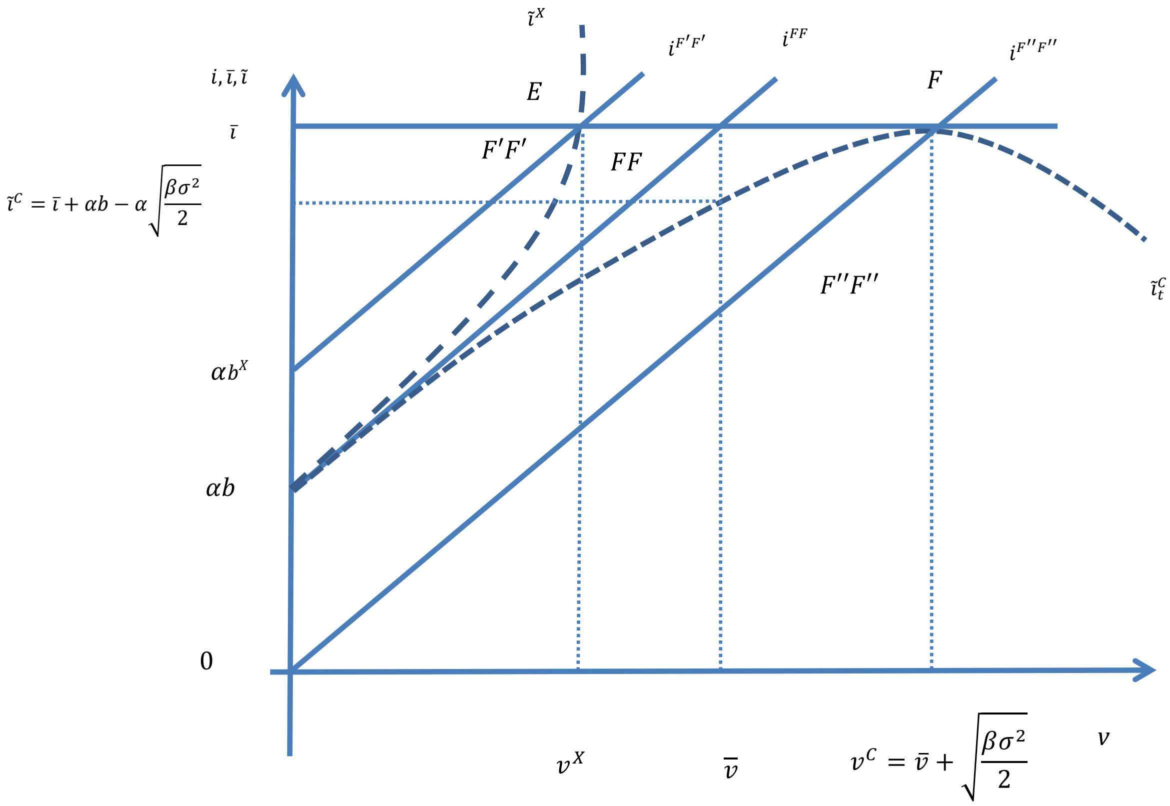

Figure 3: The interest rate reduction when moving from the ECB non-credibility interest rate path ($\tilde{\iota}^X$), to the credibility one ($\tilde{\iota}_t^C$).

3.4.3 The case of a partially credible interest rate target zone

A third situation can be considered, namely the one in which the joint effort of the monetary and fiscal authority is only sufficient to cover a fraction ρ (with $0 \leq \rho < 1$) of b, so that $VR^{PC} = \rho b$. Taking the same steps followed in Section 2.4.2 it can be concluded that since only a part of public debt can be stabilized, the size of the 'honeymoon' gets reduced:

$$v^C = v^{PC} + (b - VR^{PC}). \tag{60}$$

The case of imperfect credibility is compared with the case of stabilizing full credibility in Figure 4.

It should be noted that this is an additional result which is due to this second generation of target zone modeling, allowing to associate directly different degrees of credibility to different levels of availability of virtual reserves.

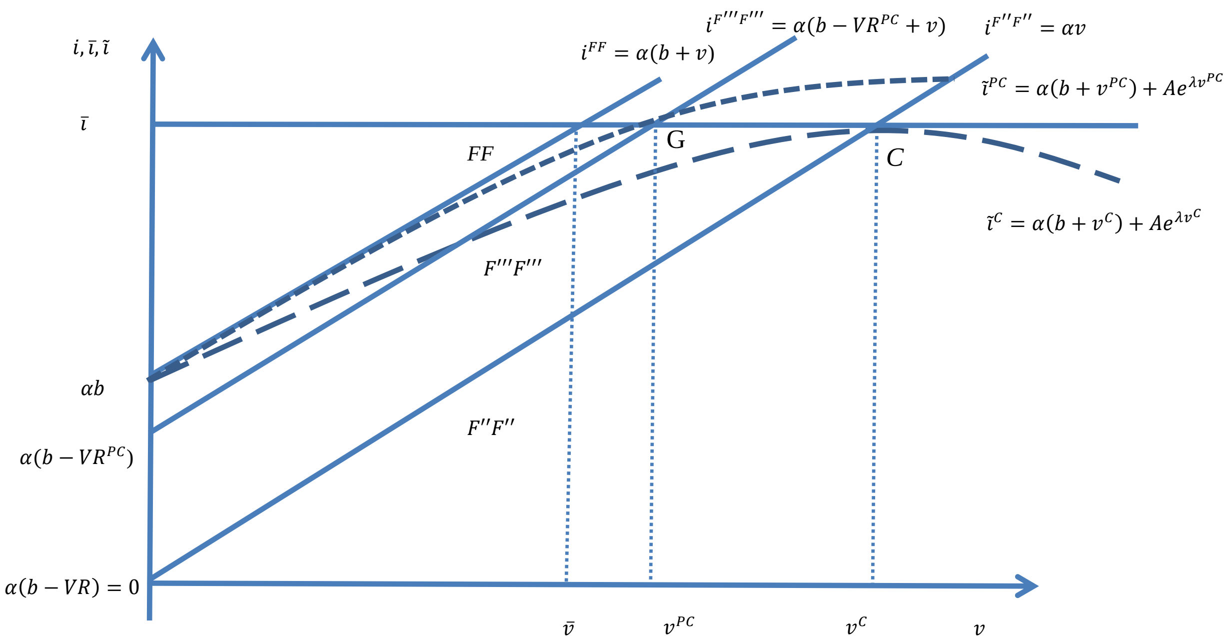

Figure 4: A fully credible ($\tilde{\imath}^C$) and a partially credible ($\tilde{\imath}^{PC}$) interest rate target zone.

It is also possible, however, to add a different – and maybe clearer - interpretation of the case of imperfect credibility. As a matter of fact, the observation that the interest rate path crosses the interest rate target without satisfying a 'smooth pasting' condition made by Della Posta (2018), does not mean that the latter does not play a role anymore. 'Smooth pasting' still plays a role, but, given the size of the shock, it does so at a higher required stability interest rate level. Such a higher interest rate level, however, would only be feasible if the domestic policymakers (monetary and fiscal authority) had enough 'ammunitions' to accommodate it, namely if the central bank could print enough money or the government could run a sufficiently large primary surplus to absorb the shock. This is where, then, becomes relevant to identify the determinants of $\bar{\imath}$, as shown in Eq. (53). Let us explain the economic intuition. The demand shock is such that public debt sustainability can only be granted if the higher interest rate charged by the market could be resisted by the policymakers. In other words, such a higher interest rate could be avoided if either m or s could be increased. As long as that is the case, i^*, namely the interest rate granting public debt stability, will *not* increase above $\bar{\imath}$, the maximum feasible interest rate that the domestic policymakers can stand in order to assure public debt stability, and the 'smooth pasting' condition will still apply. If that is not the case, however, and it turns out that $i^* = g^* + \frac{m^*+s^*}{b^*} > \bar{\imath} = g^* + \frac{\bar{m}+\bar{s}}{b^*}$, this is due to the fact that $(m^* + s^*) > (\bar{m} + \bar{s})$, namely the monetary and fiscal policy which would be necessary to stabilize public debt exceed those that are feasible. 'Smooth pasting', then, would apply in correspondence with $i^* > \bar{\imath}$, as Figure 5 shows clearly.

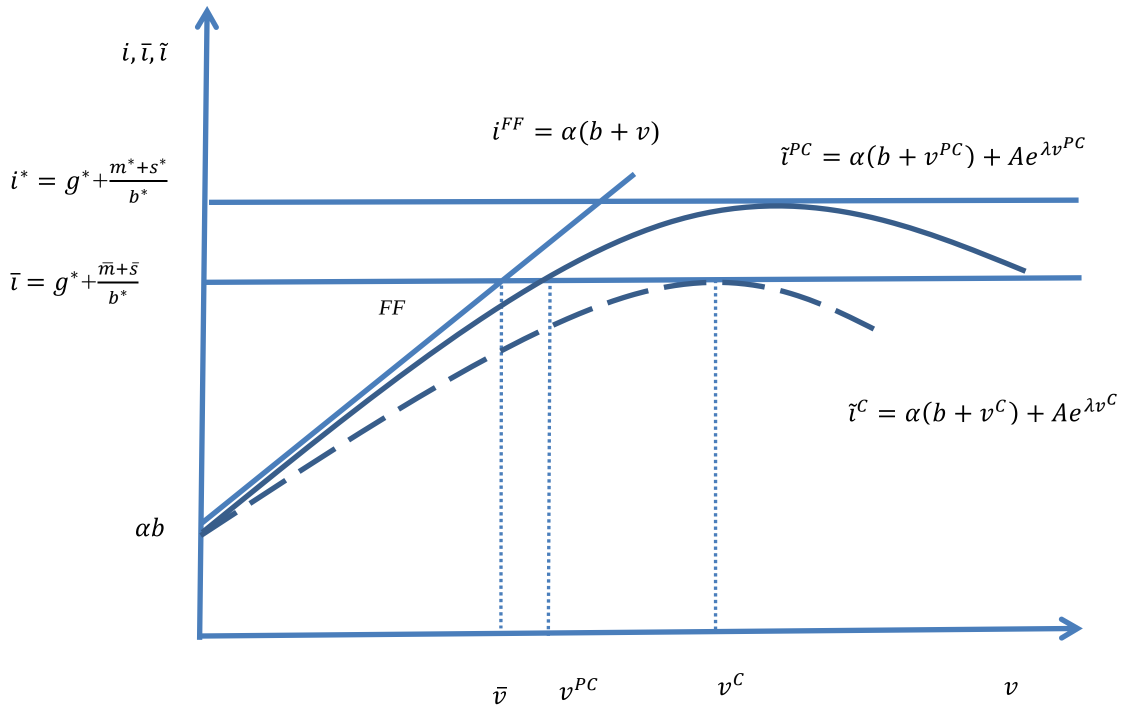

Figure 5: A graphical interpretation of the pPartial credibility of an interest target zone.

4. Target zones and the current retreat of economic globalization

The case of a government committing to protect its citizens against the costs of globalization can also be analyzed within a target zone model (see Della Posta, 2020a). The defense that can be provided by the government, however, can have different degrees of credibility, determining opposite outcomes. The apparently paradoxical result that emerges from this analysis is that the best way to let the globalization process proceed is to provide a credible defense of the citizens, making sure that the economic and social costs that they have to stand will not overtake a given credible upper threshold. If that is the case, a globalization 'honeymoon' can be enjoyed, namely it could proceed beyond the boundary that would be binding in the absence of any expectational effect. A 'divorce' from globalization occurs, instead, when the upper threshold of economic and social costs resulting from globalization is not credibly defended, for example because the preferences of the government are known to be biased in favor of globalization. In that case a convex non-linearity emerges, allowing to represent the current phase of retreat from economic globalization.

4.1 The target zone model of economic globalization

The modeling of the process of economic globalization is quite intuitive and allows to cast the problem in a standard target zone setting.

It can be argued that economic globalization, g_t produces both benefits and (true or perceived) economic and social costs, c_t. Such costs, however, have also an expectational, 'self-fulfilling' component, namely they are affected by their expected future variation, so that: [9]

(61) $$c_t = \delta g_t + \beta \frac{E[dc_t]}{dt},$$

Parameters δ and β express the sensibility of the costs with respect to the current state of economic globalization and to the expected future variation of the costs themselves.

The dynamics of the globalization process, g_t then, can be thought as represented by the following arithmetic Brownian motion equation:

(62) $$dg_t = \mu dt + \sigma dz.$$

The term μdt refers to a deterministic drift evolving at the constant rate μ, while the stochastic component is the well-known Browning motion process $\sigma dz.$

Citizens are assumed to believe that their domestic government is committed, either explicitly or implicitly, to protect them against the negative effects of economic globalization, so that:

(63) $$c_t = c^* \text{ if } c_t \geq c^*$$

[9] The equation above captures quite intuitively the fact that the costs of globalization are both real and perceived.

$$c_t = \tilde{c}_t \;\text{ if } c_t < c^*,$$

where c^* represents the upper threshold above which the costs of economic globalization will not be allowed to move and $\tilde{c}_t$ is the value of the costs when they resent the expectational effect due to the presence of a more or less credible upper target, as it will be discussed below. [10]

It could be argued, for example, that a populist government would be claiming to protect quite strictly its citizens, thereby accepting to stand a rather low cost of globalization. On the contrary, a traditional market-oriented 'center-left' or 'center-right' government would be willing to assign a higher weight to the global benefits resulting from economic globalization so as to be willing to stand a higher level of costs.

The economic globalization target zone model, then, is composed by Eqs. (61), (62), (63) and (6).

4.2 The solution in the case of a credible target on the costs of globalization

The general solution of Eqs. (61), (62), (63) and (6) (still considering only the upper band) is:

(64) $$c_t = q(g_t) = \delta g_t + \beta\mu + A_1 e^{\lambda_1 g_t},$$

where - in the case in which the upper limit for the economic and social costs of globalization is expected to be defended by the government – the, by now well-known, 'smooth-pasting' condition applies. By imposing it, it turns out that:

(65) $$A_1 = \frac{-e^{-\lambda_1 g_H}}{\lambda_1} < 0.$$

As a result:

(66) $$\tilde{c}_H = c^* = \delta g_H + \beta\mu - \frac{1}{\lambda_1} = c'_H - \frac{1}{\lambda_1},$$

Where $c'_H = \delta g_H + \beta\mu$.

And it also turns out that:

(67) $$g_H = \frac{1}{\delta}(c^* - \beta\mu + \frac{1}{\lambda_1}) = \bar{g}' + \frac{1}{\delta\lambda_1},$$

where $\bar{g}' = \frac{1}{\delta}(c^* - \beta\mu)$.

[10] In what follows we will ignore, for obvious reasons, any lower target for the costs.

The term, $\frac{1}{\delta\lambda_1} = \frac{2\sigma^2}{-\mu+\sqrt{\mu^2+2\sigma^2/\beta}} > 0$, determines the size of the 'honeymoon', namely it tells by how much the process of globalization can proceed beyond the level $\bar{g}'$ corresponding to the maximum bearable cost, c^*.

What the 'smooth pasting' condition suggests is that the more $\tilde{c}_t$ approaches c^*, due to both the deterministic drift and the stochastic shocks hitting the process of globalization, the more a government's intervention aiming at reducing g_t - implicitly made credible by the government's political preferences that are assumed to be known- is expected in such a way that $\tilde{c}_t \leq c^*$.

4.3 The 'divorce' from economic globalization

A different outcome would be obtained when it is not sure anymore that the upper limit on the cost of globalization will not be exceeded. By following once more the approach taken by Bertola and Caballero (1992) we have:

(68) $$c_t = q(g_t, m) = \delta g_t + \beta\mu + Ae^{\lambda(g_t - m)}.$$

with m being the center of the band within which globalization is moving. It also follows that:

(69) $$p[\left(\bar{g} + \frac{\delta}{2}\right) + \beta\mu + A] + (1-p)\left[\left(\bar{g} - \frac{\varepsilon}{2}\right) + \beta\mu \mp A\right] = \bar{g} + \beta\mu + Ae^{\lambda\frac{\bar{g}}{2}},$$

and:

(70) $$A = \frac{\alpha[p\left(\frac{\delta+\varepsilon}{2}\right) - \frac{\varepsilon}{2}]}{e^{\lambda\frac{\bar{g}}{2}} - 1}.$$

This also means that $A \geq 0$ iff $[p\left(\frac{\delta+\varepsilon}{2}\right) - \frac{\varepsilon}{2}] \geq 0$, that is iff:

(71) $$p \geq \frac{\varepsilon}{\delta+\varepsilon},$$

In such a case, the cost of globalization follows a convex non-linearity with respect to economic globalization, moving up more than proportionally with the latter, which is what the current phase of economic globalization is experiencing.

4.4. Heterogeneous agents in the target zone model for economic globalization

The traditional target zone modeling assumes that agents are all alike and that they are all informed about the value taken by the upper target of the variable under control. At most, as in the case considered by Bertola and Caballero (1992), they may assign a given probability to the event that the band is moved up or down. Tamborini (2015) considers, instead, in a completely different setting, the case in which agents are characterized by heterogenous beliefs about the true value of the threshold for the primary surplus and assumes, for example, a normal distribution for it. It is possible, then, to merge those two approaches. As a result, in a heterogeneous agents' model the probability that the upper target will not be overtaken (and of course its complement) can be endogenized, depending on the proportion of heterogeneous agents believing that it has been already reached and overtaken. When considering the case of 'divorce', for example, the assumption of agents' heterogeneity allows to endogenize the probability p, that Bertola and Caballero (1992) had taken exogenously. As a matter of fact, following the methodology adopted by Tamborini (2015) in the different context of economic globalization, as Della Posta (2020b) does, it is possible to argue that the probability assigned by the market to the fact that globalization has already moved beyond the level that the government was committed to defend, depends on the proportion of heterogeneous agents sharing that belief: the larger the value taken by the level of globalization, g_t, the higher the proportion of agents who believe that such a value exceeds what the government had committed to resist, thereby inducing a spike of the costs of globalization, what can be defined a 'divorce' from social stability. This means, then, that:

(72) $$F(g_t) = \int_{\underline{g}}^{g_t} f(g^*)\, dg^*,$$

where $F(g_t)$ is the cumulative distribution function of the normally distributed threshold level, g^*, included between $\underline{g}$ and $\bar{g}$, above which the government should not let the level of globalization go. As it is clear, $0 \leq F(g_t) \leq 1$ represents, then, the fraction of people according to whom $g_t > \underline{g}$, which increases with g_t. When $g_t < \underline{g}$ it turns out that $F(g_t) = 0$, namely no agent believes that the level of globalization has reached its upper limit, while when $g_t \geq \bar{g}$ all agents believe that globalization has reached its upper limit, namely $F(g_t) = 1$. According to what we have discussed above it is possible to conclude that:

(73) $$F(g_t) = p(g_t)$$

The arbitrage equation when g_t reaches the upper target g^*, then, leads to conclude that $A \gtreqless 0$ iff $[p(g_t)\left(\frac{\delta+\varepsilon}{2}\right) - \frac{\varepsilon}{2}] \gtreqless 0$, that is $A \gtreqless 0$ iff:

(74) $$F(g_t) = p(g_t) \gtreqless \frac{\varepsilon}{\delta+\varepsilon}.$$

The equation above implies that the probability that globalization will be unbounded increases endogenously with g_t (in the interval of existence of the random variable g^*, included between $\underline{g}$ and $\bar{g}$).

The sign of A, which is negative for low values of g_t, then, turns positive as soon as the latter reaches the critical value $p_{CR}(g_t) = \frac{\varepsilon}{\delta+\varepsilon}$.

As it is easy to understand, when $A < 0$ we are in the case of 'honeymoon' and globalization will reach a level g^T (the largest level that it can take) that exceeds g^*, which is the one resulting from a linear relationship with its costs. In the case of 'divorce', in which $A > 0$, instead, $g^T < g^*$.

Having merged the heterogeneous agents assumption made by Tamborini (2015) with the 'divorce' target zone model proposed by Bertola and Caballero (1992) not only allows to bridge quite nicely the results of two different and otherwise orthogonal streams of literature, but it also allows to remove the stringency resulting from target zone modeling in determining either an underestimation or an overestimation of the costs of globalization, with no explanation of the passage from one to the other. In particular, this allows explaining how globalization was embraced with favor up to the point at which it reached a level that increased the share of those who thought that it was excessive, well above the critical value that changed the sign of A from negative to positive. This is how, then, the world has moved from a globalization 'honeymoon' to the current phase of retreat from globalization.

Figure (6) (as in Della Posta, 2020b) gives an intuitive idea of the conclusion that is reached by merging the two different streams of literature. It compares the linear relationship between $g^*$and c^* (when expectations play no role), with the nonlinearity resulting from the expectation of an upper intervention to avoid globalization (and its costs) to exceed a given upper threshold. Due to the heterogeneity of agents, though, A moves from negative to positive values so that the initial stabilizing effect of the expectation of such an intervention (the 'honeymoon') gradually fades away when globalization proceeds and, as a result, the costs increase, giving way, after a critical level has been overtaken ($g_t = 10$ in the simulation represented in Figure 6; see Della Posta, 2019 for further details), to a 'divorce'.

5. Concluding remarks

Krugman's 'honeymoon' paper showed the stabilizing properties of the adoption of floating bands and initiated the literature on exchange rates target zones. The contributions that followed, however, introduced the opposite idea of a destabilizing 'divorce' (Bertola and Caballero, 1992), and merged that literature with the one on speculative attacks on fixed exchange rates, to underline the role played by foreign reserves in determining the different degrees of credibility of an exchange rate target zone (Krugman and Rotemberg, 1992).

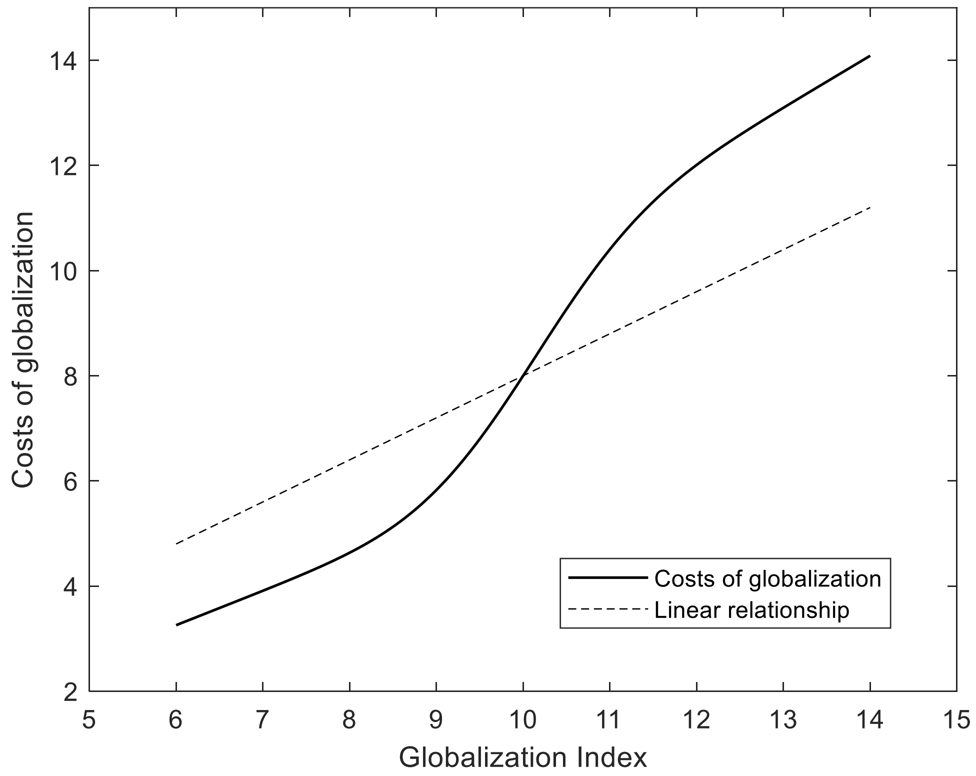

Fig. 6 – 'Honeymoon' and 'divorce' in the process of globalization

The recent euro area crisis on public debt revived that literature, producing what I define in this paper a second generation of target zone modeling, which is based on the idea that the sustainability of public debt also implies the adoption of a target, which is relative, however, to interest rates rather than exchange rates. The lack of credibility of such a target (resulting, for example, from the fact that the central bank does not operate as a buyer of last resort, as pointed out by De Grauwe, 2012 for the euro area), explains the interest rates non-linearity that was observed during the euro area crisis (as documented by De

Grauwe and Ji, 2013a) and allows to understand why some non-Eurozone countries have not been subject to speculative attacks in spite of public debt-to-GDP ratios that were as large as those of crisis countries, but that were guaranteed by the 'virtual' reserves of their domestic central bank (De Grauwe, 2012). The 'divorce' emerges, then, also in this literature, together with the role played by the 'virtual' reserves available with both the central bank and the government: any credibility bonus (what Krugman, 1991, dubbed as 'honeymoon' effect), is not a gift from heaven but, with rational agents, it reflects the evaluation of the size of reserves that are available with the central bank.

In applying the exchange rates target zone literature to the different context of the public debt euro area crisis, however, some generalizations and improvements of the previous models arise, as it is often the case when a tool is applied to a different endeavor and, by doing so, new features and results emerge. In particular, there is no reason to expect that the width of the new implicit band for public debt and interest rates after the speculative attack will be as large as the previous one, as assumed instead by Bertola and Caballero (1992) in the case of exchange rate target zones. Bertola and Caballero's (1992) 'divorce' conclusion, then, could be reconsidered within this more general case.

More extensions would be possible, for example considering the fact that the interest rate target zone may be made credible thanks to the intervention of a domestic or even a federal government, in addition to the central bank. If that is the case, the interest rate target may be respected – and, as a result, public debt will be fully stabilized - even in the case in which the central bank is not operating as a buyer of last resort.

Finally, a third generation of target zone modeling applies the same techniques to analyze the process of economic globalization, that has been characterized by a first phase of 'honeymoon' and the current case of 'retreat', in which the costs of globalization are exhibiting a convex non-linearity.

More results are awaiting to be borrowed from the previous literature on exchange rates target zones, applying them to the second and third generation of target zone modeling, and new domains of application are awaiting to be found.

References

Bertola, G., and R. Caballero (1992), "Target Zones and Realignments", *The American Economic Review, 82*(3), pp. 520-536. Retrieved from http://www.jstor.org/stable/2117319

Bertola, G. and A. Drazen (1993), "Trigger Points and Budget Cuts: Explaining the Effects of Fiscal Austerity", *American Economic Review*, Vol. 83, No. 1, Pages 11 - 26.

Corsetti, G., K. Kuester, A. Meier, and G. J. Müller (2014), "Sovereign risk and belief-driven fluctuations in the euro area", *Journal of Monetary Economics*, Vol. 61, Pages 53–73, January.

De Grauwe, P. (2012), "The Governance of a Fragile Eurozone." *Australian Economic Review,* 45 (3), pp. 255–68.

De Grauwe, P. and Y. Ji (2013a), "Self-fulfilling crises in the Eurozone: An empirical test", *Journal of International Money and Finance* - The European Sovereign Debt Crisis: Background & Perspective, Vol. 34, April, pp. 15–36.

De Grauwe, P. and Y. Ji (2013b), "From panic-driven austerity to symmetric macroeconomic policies in the Eurozone", *Journal of Common Market Studies*, Vol.51, Annual Review, p. 31-41.

Della Posta, P. (2019), "Interest rate targets and speculative attacks on public debt", *Macroeconomic Dynamics*, Vol. 23, N. 7, pp. 2698-2716, October, https://doi.org/10.1017/S1365100517000931, Published online: 16 March 2018,

Della Posta, P. (2018), "Central bank intervention, public debt and interest rate target zones", *Journal of Macroeconomics*, Vol. 56, June, Pages 311-323, June. 10.1016/j.jmacro.2018.04.001.

Della Posta, P. (2020a), "The economic and social costs of globalization: a target zones analysis", *The World Economy*, First published, 17 July 2020, https://doi.org/10.1111/twec.13008

Della Posta, P. (2020b), "An analysis of the current backlash of economic globalization in a model with heterogeneous agents", *Metroeconomica*, First published: 02 September 2020, pp.1-20, 2020, DOI: 10.1111/meca.12312.

Duarte, A. P., J. Andrade and A. Duarte (2013), "Exchange Rate Target Zones: A Survey of the Literature", *Journal of Economic Surveys*, Vol. 27, Issue 2, pp. 247-268

Kempa, B. and Nelles, M. (1999), "The Theory of Exchange Rate Target Zones", *Journal of Economic Surveys*, Vol. 13, pp. 173-210. doi:10.1111/1467-6419.00081

Krugman, P. (1991), "Target Zones and Exchange Rate Dynamics", *The Quarterly Journal of Economics*, Vol. 106, No. 3. (August), pp. 669-682.

Krugman, P. and J. Rotemberg (1992), "Speculative Attacks on Target Zones", in Krugman, P. and M. Miller (eds), *Exchange Rate Targets and Currency Bands*, pp. 117-132, Cambridge University Press.

Krugman, P. and M. Miller (eds) (1992), *Exchange Rate Targets and Currency Bands*, Cambridge University Press.

Obstfeld, M. (1986), "Rational and Self-fulfilling Balance-of-Payments Crises", *American Economic Review*, 76 (1), 72-81.

Tamborini, R. (2015), Heterogeneous Market Beliefs, Fundamentals and the Sovereign Debt Crisis in the Eurozone, *Economica*, Volume 82, Issue s1, December 2015, Pages 1153–1176, DOI:

10.1111/ecca.12155.